Difficulties in interpreting Archimedes principle for objects floating in water

Dificultades para interpretar el principio de Arquímedes de objetos que flotan en el agua

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Abstract
The buoyant force on a body is defined as the net vertical force that stems from the fluid or fluids in contact with the body. According to the definition, question might be raised whether the fluid(s) inside the body should be considered as a force contacting with the body or a part of the body? The well-known Archimedes' principle to pursue buoyancy only thinks of the weight of fluids displaced but not the status inside the body itself. To address the issue, the article illustrated simulation experiments and mathematical analysis to demonstrate the buoyant force on a body with fluid inside and conclude that the Archimedes' principle is one of special cases. To stick on the definition of buoyant force, the fluid(s) inside the body should be considered as a force contacting with the body but not a part of the body.

Key words: *buoyant force, Archimedes principle, difficulties, objects floating in water.*

Resumen
Se define la fuerza flotante en un cuerpo, como la fuerza vertical neta que proviene del fluido o fluidos en el contacto con el cuerpo. Según la definición, la pregunta podría plantearse: ¿el fluido (s) dentro del cuerpo debe ser considerado como una fuerza que hace contacto con el cuerpo o una parte del cuerpo? El principio de Arquímedes se concentra en el peso de fluidos desplazados, pero no sobre el estado dentro del propio cuerpo. Para dirigirse el problema, el artículo muestra la simulación experimental y análisis matemático para demostrar la fuerza flotante en un cuerpo con el fluido dentro y concluir que el principio de ARQUÍMEDES es uno de esos casos específicos. Para definir la fuerza flotante, el fluido(s) dentro del cuerpo debe ser considerado como una fuerza que hace contacto con el cuerpo pero no con una parte del cuerpo.

Palabras clave: *fuerza flotante, el principio de Arquímedes, dificultades, objetos flotantes en el agua.*

INTRODUCTION

The well-known buoyant force (or 'up-thrust') on a body is defined as the net vertical force that stems from the fluid or fluids in contact with the body. For a body either completely submerged in one fluid or at the interface of two immiscible fluids, the buoyant force is equal to the weight of fluids displaced, which is the familiar Archimedes' principle. The derivative of the Archimedes' principle is quite straightforward and can be found in textbooks (SHAMES 1982; FOX, MCDONALD 1978; STREETER, WYLIE 1981). However, it should be stressed that the properties of the body itself (except for its volume) do not play any role during the deduction of ARCHIMEDES' principle to pursue buoyant force.

It would be interesting to know how the buoyant force changes for a floating body as it sinks lower into water. A similar question is 'How does the buoyant force on a submarine changes as it moves from a harbor and submerges into the sea?', which is frequently asked in Physics classes by junior high schools or even high schools teachers. Before further discussion, we will discuss two simulation experiments.

SIMULATION EXPERIMENT

A large glass and another one with smaller size were prepared for the experiment. Water was added to both of them so that the smaller one could float vertically on the water surface of the large one. For the first experiment, we filled additional water into the smaller cup and recorded the water level of the big glass. From the experimental data, we found that water level in the large glass increased when the supplementary water added into smaller glass. However, for the second experiment, we drew the water from inside the large glass to add into the smaller one and found that water level inside the large glasses does not change.

DISCUSSION

Suppose that the weight of the smaller glass is W_{glass}, and the weight of water inside the smaller glass is W_o. Since the small glass is floating, static equilibrium exists and the Archimedes' principle, the buoyancy, F_b, on the smaller cup before water added is,

$$F_b = W_{glass} + W_o = \gamma V_o \tag{1}$$

where γ is the specific weight of water displaced in the larger vessel (It is not necessary that the density of the substance added be the same as water – indeed any suitable 'weight' could be used to cause the smaller container to sink lower) and V_o is the volume displaced by the smaller cup. Similarly, after the addition of water weighing ΔW, the buoyant force, F_b', is,

$$F_b = W_g + W_o + \Delta W = \gamma V_o \tag{2}$$

where V_o' is the volume displaced by the smaller glass after water added. Obviously, the buoyancy force on the smaller glass increases when water added into it in line with the increased volume submerged. The analysis at equilibrium relates to the first simulation experiment. In other words, when directly using the Archimedes' principle, the buoyant force equals to the weight of fluid displaced, the buoyancy force acting on a body increases as its submerged volume increases.

It seems quite straightforward get the conclusion. However, let us go back to the derivation of the buoyancy for further investigation. As shown in figure 1, the smaller cup floats steadily on the water surface of the large glass.

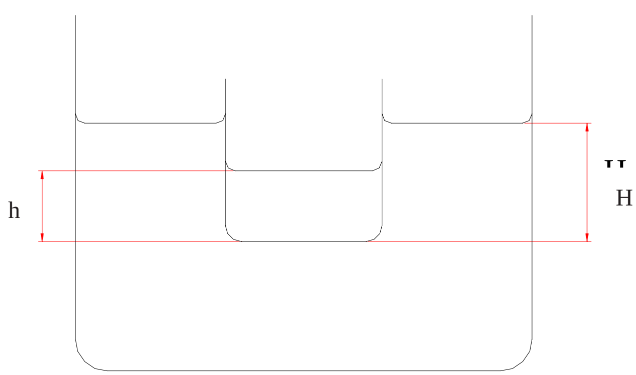

Figure 1. The smaller cup floats vertically on the water surface of the large glass

The definition of buoyant force is the net upward vertical force that stems from the fluid or fluids in contact with the body. The net upward vertical force exerted by the water in the large glass, F_v exert on the bottom of the smaller glass is,

$$F_v = \gamma HA - \gamma hA = \gamma A(H - h) \tag{3}$$

where H is the height from the bottom of smaller glass to the water surface in larger glass, h is the height of water inside the smaller glass, and A is the cross-section area of the smaller glass. For static equilibrium, the net upward vertical force must be equal to the weight of the smaller glass, which means

$$F_v = \gamma A(H - h) = W_{glass}\,. \tag{4}$$

Since W_{glass}, the weight of the smaller glass is a constant value, therefore the net upward vertical force, F_v, is also a constant. Therefore, according to the definition of buoyant force, the net upward vertical force that stems from the fluid or fluids in contact with the body, the buoyant force acting on the smaller glass should be unchanged, which is certainly a contradiction as the point view from Archimedes principle's. However, the conclusion will be certainly questioned 'why should the buoyancy be unchanged as the small glass displaced more water?'

As we know that $W_o = \gamma Ah$, the weight of water inside the small glass, the equation 4 can be further expressed as

$$W_{glass} + W_o = \gamma AH = \gamma V_o\,, \tag{5}$$

which is exactly the same as equation 1. In other words, the Archimedes' principle is one of the formulas to calculate the buoyancy. At first appearance, the weight inside the small glass, W_0, increases so the volume displaced by the small glass, V_0, also raises. Therefore, the buoyant force acting on the smaller glass should augment. However, directly counting the buoyancy with specific weight of the fluid(s) multiplying the volume displaced, the Archimedes' principle, might violate the definition of buoyant force (or 'up-thrust') on a body, if there is fluid(s) inside the body.

For the example shown in the second simulation, the water was drew from the large glass and then added to the smaller glass. After replenishing water into smaller glass, H' is the new height from the bottom of smaller cup to the water surface in larger glass and h' is the new height of water inside the smaller glass. Similar to equation 4, the net upward vertical force is

$$\gamma A(H' - h') = W_{glass}\,. \tag{6}$$

The new weight of water inside the cup is the sum of the weights of original amount and the replenishment, ΔW. Therefore, the equation 6 can be further expressed as

$$\gamma AH' = \gamma Ah' + W_{glass} = \gamma Ah + \Delta W + W_{glass}\,. \tag{7}$$

Comparing equation 7 with 5, the difference between the weight of fluid displaced by the cup before and after replenishment, γ A(H' – H), is due to the ΔW, which drew from the large glass. In other words, the water level of the large glass remains the same, which is exactly the finding of the second experiment.

To stick on the definition of buoyant force equal to the net vertical force that stems from the fluid or fluids in contact with the body, the liquid (if any) inside the body should be counted as the downward effect of fluid(s) but not the weight of the body during calculating buoyancy. The explanation would tell that the buoyant force on a body found directly from the Archimedes' principle could be misinterpreted. Although air is one of the fluids, its specific weight is generally neglected (Shames 1982). Therefore, the answer for the change of the buoyant force on a submarine from a harbor into the sea would be null in that the water pumped into it is more like the second simulation experimentation. And so it would be if the air was allowed to escape rather than being compressed into an internal tank.

From the above discussions, the fluid(s) inside a body does (do) play an important role during the deduction of the buoyant force. However, the fluid(s) inside the floating object is (are) frequently only counted as a part of the total weight of the floating body, as indicated by most of the textbooks (HECHT 2000; BENSON 1996; SERWAY 1996).

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Received 31.10.2003, **approved** 20.03.2004